NUMERICAL INVESTIGATION OF STABILITY OF A SUPERSONIC LAMINAR BOUNDARY LAYER

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We investigate the stability of a supersonic boundary layer in relation to small two-dimensional disturbances in the Mach number range $1 < M < 4$. The behavior of the neutral curves corresponding to the additional unstable frequencies discovered in [1] is examined at different Mach numbers M and surface temperatures T_W. Regions in the T_W, M plane are classified according to the number and kind of neutral curves. We show that the destabilizing effect of viscosity on the boundary layer disappears with increase in Mach number.

1. In an investigation of the stability of a supersonic boundary layer by numerical integration of inviscid equations Mack discovered the existence of higher unstable natural frequencies [2]. They appear in the case where the phase velocity c_r of an infinitely small disturbance exceeds the velocity of sound a_W at wall temperature [2]:

$$c_r / a_w > 1 \tag{1.1}$$

For a heat-insulated surface this condition is fulfilled when $M > 2.2$. Mack investigated the higher unstable frequencies which appear in this case by numerical integration of the inviscid and complete stability equations. Some results of his investigations are given in [3]. Instability of this type is caused by inertial effects, and the viscosity has a stabilizing effect.

When the surface cools, the increased unstable frequencies of this type can be significant at Mach numbers close to unity if the cooling is rapid enough and condition (1.1) is fulfilled.

A determination of the conditions for complete stabilization in [1] showed the existence of additional stable frequencies. These frequencies exist in ranges of surface temperatures at which condition (1.1) is not fulfilled. At Mach numbers close to unity the instability of these additional frequencies is due to viscosity.

In the present paper we investigate the behavior of the neutral curves corresponding to the additional unstable frequencies discovered in [1] in relation to change in Mach number M and surface temperature T_W.

2. The stability is investigated by numerical integration of the Lees–Lin equations

$$\begin{gathered}
i (U - c) r + \rho (\varphi_y + if) + \rho_y \varphi = 0 \\
\rho [(i (U - c) f + U_y \varphi] = -\frac{i\pi}{\gamma M^2} + \frac{\mu}{\alpha R} [f_{yy} + \alpha^2 (i\varphi_y - 2f)] + \\
+ \frac{2}{3} \frac{\mu_2 - \mu}{\alpha R} \alpha^2 (i\varphi_y - f) + \frac{1}{\alpha R} [sU_{yy} + sU_y + \mu_y (f_y + i\alpha^2 \varphi)] \\
\rho [i (U - c) \varphi] = -\frac{1}{\alpha^2} \frac{\pi_y}{\gamma M^2} + \frac{\mu}{\alpha R} (2\varphi_{yy} + if_y - \alpha^2 \varphi) + \\
+ \frac{2}{3} \frac{\mu_2 - \mu}{\alpha R} (\varphi_{yy} + if_y) + \frac{1}{\alpha R} \left[isU_y + 2\mu_y \varphi_y + \frac{2}{3} (\mu_{2y} - \mu_y)(\varphi_y + if) \right] \\
\rho [i (U - c) \theta + T_y \varphi] = -(\gamma - 1)(\varphi_y + if) + \frac{\gamma}{\alpha R \sigma} [\mu (\theta_{yy} - \\
- \alpha^2 \theta) + (sT_y)_y + \mu_y \theta_y] + \\
+ \frac{\gamma (\gamma - 1)}{\alpha R} M^2 [sU_y^2 + 2\mu U_y (f_y + i\alpha^2 \varphi)] \\
\pi = r/\rho + \theta/T
\end{gathered} \tag{2.1}$$

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with boundary conditions

$$f(0) = \varphi(0) = \theta(0) = 0, \quad f, \varphi, \theta \to 0 \text{ as } y \to \infty \tag{2.2}$$

Here U, ρ, and T are the time-averaged velocity, density, and temperature; f, $\alpha\varphi$, r, θ, π, and $s = \theta\, d\mu/dT$ are disturbances of the longitudinal and transverse velocities, density, temperature, pressure, and viscosity; y is the distance along the normal to the surface; the subscript y denotes differentiation; γ is the adiabatic constant; M is the Mach number; σ is the Prandtl number; μ and $\mu_2 = -{}^2/_3\,\mu$ are the viscosity coefficients; R is the Reynolds number; α is the disturbance wave number; and $c = c_r + ic_i$ is the disturbance phase velocity. We investigate neutral disturbances and, hence, $c_i = 0$. We assume that the dependence of the disturbances on the longitudinal coordinate and the time has the form exp $i\alpha(x-ct)$.

To reduce system (2.1) to a form suitable for numerical integration we introduct the variables

$$z_1 = f, \; z_2 = f_y, \; z_3 = \varphi, \; z_4 = \pi/\gamma M^2, \; z_5 = \theta, \; z_6 = \theta_y$$

The substitution reduces system (2.1) with boundary equations (2.2) to the form [4]

$$z_{iy} = \sum_{j=1}^{6} C_{ij} z_j \quad (i=1,\ldots,6) \tag{2.3}$$

$$z_1(0) = z_3(0) = z_5 = 0 \qquad z_1, z_3, z_5 \to 0 \text{ when } y \to \infty \tag{2.4}$$

The values of α, R, and c_r required for satisfaction of the boundary conditions (2.4) can be sought by the method described in [5]. Outside the boundary layer we seek analytical solutions, decaying at infinity [4], which are integrated numerically from the upper limit to the wall by the orthogonalization method [6]. From the obtained special solutions at y = 0 we plot the amplitude of the thermal disturbance as a function of α, R, and c_r and seek the zero of this function by Newton's iteration method [4].

As practice in calculations showed, this is possible only in the case where the values of c_r on the upper and lower branches of the neutral curve differ considerably. As the surface cools, $c_r \to 1 - M^{-1}$ on the upper and lower asymptotes. Hence, beginning at a certain value of R, the search for eigenvalues does not lead to convergence.

In the present paper we regard the amplitude of the thermal disturbance on the surface as a function of α, αR, and $\sqrt{1-M^2(1-c)^2}/\alpha$. These parameters vary smoothly along the neutral curve and on the asymptotes αR and $\sqrt{1-M^2(1-c)^2}/\alpha$ level out to different constant values. This enables us to calculate to $\alpha = 0$ and, using the method of [1], to find the conditions for complete stabilization of the investigated neutral curves.

In the numerical calculations we took $\sigma = 0.72$ and $\gamma = 1.41$. We assumed that the viscosity varied with temperature in accordance with Sutherland's law

$$\mu = (T)^{3/2} \frac{1 + (T_s/T_\infty)}{T + (T_s/T_\infty)}$$

where $T_S = 110°$ is the Sutherland constant, and $T_\infty = 157°$ is the temperature at infinity. The eigenvalues for M = 2.2 were calculated at this temperature in [4] and are compared with some of the results of the present work.

For the determination of U, T, and ρ — the free-stream velocity, temperature, and density distributions — we numerically integrated the equations of the laminar boundary layer on a flat plate in conjunction with system (2.3). The integration was performed by the method described in detail in [4].

3. We investigated the stability in the range 1 < M < 4. In Fig. 1 the complete stabilization temperatures T_w^* of the first (1) and second (2) neutral curves are plotted as a function of M. The dashed line denotes the temperatures of formation of these neutral curves. If we limit ourselves to M = 3.3 the intersections of the curves in Fig. 1 form five regions. The neutral stability curves characteristic of each region are shown in Figs. 2 and 3. The letters beside the neutral curves denote the regions to which these curves correspond.

In region a there is one neutral curve of complex form. For M = 2.2 there are two such curves (Fig. 2, $T_w = 1.837$ — the heat-insulated surface, and $T_w = 1.75$). For M = 3 there is one neutral curve (Fig. 3, $T_w = 2.3$). The dots in Fig. 2 denote the points, taken from [4], for the heat-insulated surface.

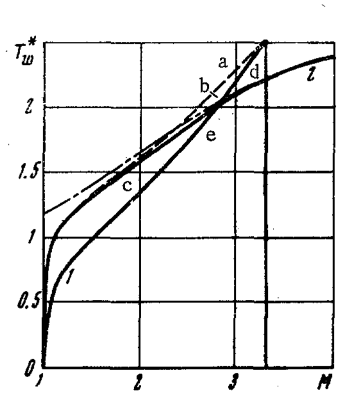

Fig. 1

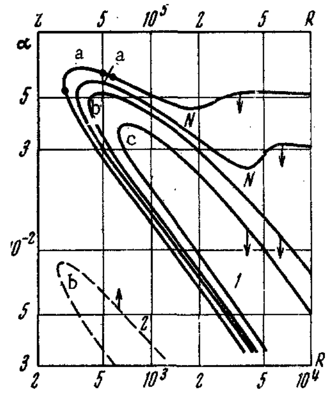

Fig. 2

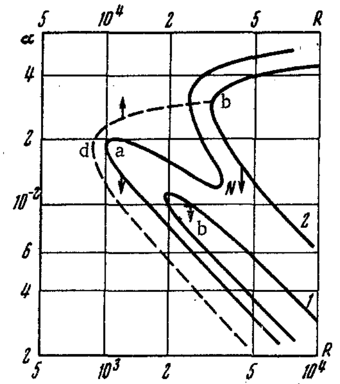

Fig. 3

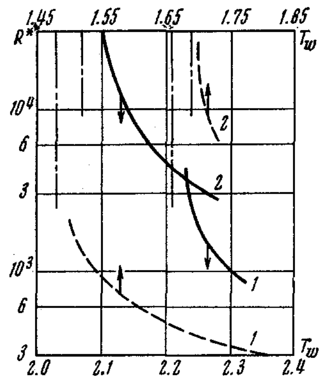

Fig. 4

With reduction of surface temperature the neutral curve is deformed so that for point N (Figs. 2 and 3) $\alpha \to 0$, $R \to \infty$. This leads to the formation of two neutral curves, which exist in region b (Fig. 2, M = 2.2, T_W = 1.7; Fig. 3, M = 3, T_W = 2.25). The top scale is used to obtain the dashed curves in Figs. 2, 3, and 4.

In region c there is only the first neutral curve (Fig. 2, M = 2.2, T_W = 1.6), and in region d only the second (Fig. 3, M = 3, T_W = 2.15).

Region e is the region of complete stabilization.

According to the foregoing, all the neutral stability curves can be divided into three kinds and on the basis of the presented results we can determine the role of viscosity in the stability of the supersonic boundary layer.

Curves of the first kind exist in regions b and c. Examples of such curves are shown in Fig. 2 (continuous curves b and c, dashed curve b) and Fig. 3 (lower curve b). A characteristic feature of these curves is that with increase in wave number the Reynolds number decreases on both branches of the neutral curve, and the highest α is attained close to the critical Reynolds number. Such neutral curves are characteristic of instability due to viscosity.

Curves of the second kind exist in regions b and d. Examples of such curves are shown in Fig. 3 (upper curve b and dashed curve d). The highest wave number for these curves is attained on the upper asymptote, and with reduction in R the wave number on the upper asymptote is reduced. In this case the instability is due to inertial (inviscid) effects, and the viscosity has a purely stabilizing effect.

For neutral curves of the third type the instability is due to inertial and viscid effects. Such curves have an inviscid asymptote, but the highest α is attained near the critical Reynolds number. Curves a in Fig. 2 are examples of such curves.

Applying the proposed classification to the neutral curves responsible for the stability of the supersonic boundary layer we can see that the first neutral curve 1 always belongs to the first type, i.e., is due to viscosity. The second neutral curve 2 belongs to the first type at low Mach numbers. With increase in

M at temperatures above the temperatures for complete inviscid stabilization (dot-dash curve in Fig. 1) an inviscid asymptote appears on it, i.e., it belongs to the third type, while in region d it belongs to the second type.

An increase in Mach number alters the role of the neutral curves in boundary layer stability. At low Mach numbers the first curve 1 plays the decisive role in stability. Figure 4 shows the values of the critical Reynolds numbers R^* as a function of the surface temperature T_W. For $M=2.2$ (dashed curves) the critical Reynolds numbers of the first curve 1 are always less than those of the second curve 2. With increase in M and as inviscid effects begin to predominate at the second neutral curve it begins to play the leading role in stability. For $M=3.0$ the stability is determined either by the first or by the second neutral curve, depending on T_W (Fig. 4, continuous lines), and when $M>3.3$ it is determined only by the second neutral curve.

Thus, the destabilizing effect of viscosity on the boundary layer disappears with increase in Mach number. This is manifested in the reduction of the role of the first, purely "viscid" neutral curve and in assumption of predominance by inertial effects at the second neutral curve.

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